Analysis of ordered random values is an important subject in statistics and applied probability. The sample extremes are natural indicators of the range of random phenomena, whereas the sample median is a precise estimate of central observations, more reliable than the mean. Order statistics and their linear combinations constitute a simple, but very useful family of functions of observations, widely used in statistical inference. If time-valued random phenomena are examined, and the experiment time is restricted, then only some smallest order statistics are recorded. There are known various censoring models. We mainly focus on a general and flexible scheme of progressively censored type II order statistics: a large number of objects primarily undergoes a lifetime measuring experiment, but only a small number of failures is observed. The remaining objects are divided into groups and after a failure some randomly selected still living objects are removed from the study. Also, order statistics appear to be useful in examining the lifetime distributions of technical systems composed of identical elements. Then the system lifetime distribution is a mixture of distributions of order statistics of component lifetimes with coefficients depending merely on the structure of the system. The record values are also popular examples of ordered random data. They are important in analysis and prediction of catastrophes, natural disasters, and other extreme phenomena. They appear quite rarely, though, and so their much more frequent generalizations, *k*th extremes called *k*th record values, are applied in statistical studies.

For years, order and record statistics were examined separately, although they share many properties. A successful attempt to build a model combining both these notions and many others was done by Kamps (1995). He defined generalized order statistics whose multivariate distributions are constructed on the basis of a single one-dimensional distribution and a number of positive parameters. For specific choices of parameters, we obtain the multivariate distributions of consecutive order and record statistics as well as the observations of the progressively censored model. This notion is flexible enough to represent order statistics and records in specific real-life problems when the failures of consecutive objects disturb further lives of still-living ones. The generalized order statistics model is the main subject of our investigations. We first aim at developing the distributional theory of generalized order statistics under possibly least restrictive constraints on their parameters. There is a significant gap between the developments of the theory in the general model and in its submodel called *m*-generalized order statistics where the number of parameters is reduced to two. Analysis of the general model is far more complex, and requires tools of various branches of mathematics, e.g., the theory of special functions and complex analysis. A special emphasis will be laid here on discrete distributions which are of vital importance in practice. They appear naturally if the resistance of objects under study is gauged by the number of its short-time uses till the failure, and when the experiment observation cannot be performed continuously, but its inspection is possible merely at some fixed moments of time. Analysis of ordered random variables coming from discrete populations is much more sophisticated than in the continuous case, and needs more elaborate and delicate tools of reasoning. For large sets of ordered data, precise description of probability distribution is hardly tractable, and asymptotic approximations become useful. We analyze asymptotic distributions of functions of order statistics when the standard assumptions are relaxed. We admit independent observations with possibly different distributions, and some weak violations of independence condition, including stationarity and other notions of weak dependence.

Another topic of the project is a characterization of distributions based on ordered observations. The problem consists in proving that a given property of random variables uniquely describes a narrow family of parent distributions. Characterization theorems have various applications in statistical inference, e.g. goodness-of-fit tests which allow to verify whether the random data come from a specific family of distributions. Typical problems we plan to copy with are the characterizations of the following type: if the expectation of some function of ordered variable under condition that another one takes on a fixed value is expressed by a given functional relation, then the parent distribution is an element of a particular family. We further focus on establishing optimal bounds on the moments and quantiles of ordered variables and their functions for various wide families of parent distributions. We aim at providing sharp bounds on the expectations of generalized order statistics coming from distributions with increasing density and failure rate functions and evaluating the variances of some linear functions of order and record statistics by means of the population variance units. We also estimate their quantiles in various scale units. Some inequalities for linear combinations of order statistics can be applied in estimating the expected lifetimes of systems composed of identical elements. This is not possible in the variance case, though. Recently, team members proved tight evaluations of lifetime variances for systems composed of identical items and extended them to a very general family of dispersion measures. Our purpose is now to deliver analogous bounds for wide classes of location measures. Comparisons of moments of system and component lifetimes were possible owing to mixture representations of their distributions. That was an incentive for comparing moments of general mixtures of ordered random variables. The results will be applied in global analysis of Bayesian and regression models as well as in specific problems of actuarial mathematics.