In this project we will consider Lèvy risk processes. They describe the behaviors that are inherently random or whose big deterministic complexity makes just such modeling is most appropriate. Wherefore Lèvy processes have many applications, for example in financial mathematics and insurance.

We plan to consider so-called Parisian delay. This is specified in advance deterministic time d> 0, through which the Lévy process must be in a defined position: for example, above / below a certain level (so-called barrier). By applying Parisian delay some issues (such as dividend payments) become much more mathematically complex, and often better reflects the reality. The name for this problem is borrowed from the Parisian option. Depending on type of such option the prices are activated or canceled if underlying asset stays above or below barrier long enough in a row. In the project we will consider standard Lévy process and also reflected at the different barriers Lévy process. The project aim is to create the general theory of Lévy processes, refracted and reflected Lévy processes with Parisian delay implementation.

In addition, the results obtained in the project will help in a deeper examination of the impact of the Parisian delay on actuarial characteristics associated with different functionals, types of ruin and the Gerber-Shiu function.

Classic ruin is the first time when financial surplus process, modeled by a Lévy risk process becomes negative. Additionally, we can enrich the analysis of considerations relating to the so-called discounted Gerber-Shiu function, which contains a moment of bankruptcy of surplus process, the surplus immediately before the moment of ruin and the deficit at the moment of ruin. It is known that the Gerber-Shiu function is a powerful analytical tool that has wide applications in insurance and finance.

In this project we will consider Parisian delay that is a generalization of the classical approach. We can consider this delay in different scenarios, for example, at dividend payments or at the moment of ruin. In the case of ruin, this means that we will talk about bankruptcy if the surplus process remains negative for longer than the fixed d> 0. We believe that Parisian ruin probability could be a better measure of risk in many situations giving possibility for insurance company to get solvency. As it turns out, such a policy is pursued in many countries, including the United States, France and Japan.

In particular the directions of our studies will be focused on, dependent on each other, topics:.

1. Integral functionals with Parisian delay implementation.
2. Occupation times of intervals for spectrally negative Lèvy processes with Parisian delay implementation.
3. Parisian exit problems of γ –reflected Lèvy process.
4. Refracted Lèvy process with Parisian delay implementation.

The analysis of the above issues, which will be examined in this project, has been the subject of active studies in recent years. Motivations are delivered by a number of open theoretical problems as well as applied probability models in fields such as risk theory, queuing theory and financial mathematics.

Among others, a crucial role is played by analysis of dividend payments with various additional conditions (such as "taxes") and also finding the ruin probability with Parisian delay or with so-called "lower ultimate bankrupt barrier" and its asymptotic in one-dimensional and multidimensional models. All these issues will be considered in the project, so the results obtained will have a significant contribution to the general risk theory of Lèvy processes.