

Reasons for Choosing the Research Topic

Computer assisted proof development facilitates checking the correctness of reasonings that are developed in mathematical sciences. The use of computer forces the user to analyze all the proof steps and cases in great detail, that ensures that small errors (but not only these ones) are avoided.

The possibility of verifying the existing mathematical proofs, at such a high formal level of trust, was the starting point to build proof-assistants that allow computer verification of mathematical proofs, e.g., Coq, Isabelle, HOL Light, Mizar. The effective exploitation of such formal system requires developed, certified, sufficiently large libraries of mathematical knowledge stored in a formalized way as well as the possibility of automatic data migration between the largest of them.

This migration support the process of formalization providing the ability to use the formalized theorems of several libraries, but also independent verification of deductions in one system that are collected in the library of another system. The lack or inaccuracy of such migration mechanism substantially isolates a system from the other, as in the case of Mizar system, developed mainly in Poland, which has one of the world's largest databases of formalized knowledge (MML) including many domains that have not been formalized elsewhere.

Objective of the Project

The main scientific goal of this project is to provide an independent verification of the MML proofs scripts in the Isabelle logical framework. First, we will express the Mizar environment, i.e., the language and the meta-logic basing on the Isar inner syntax notation and defining new objects of logic in Isabelle/FOL, respectively, as well as we reconstruct Mizar automatization and mechanism of proof verification in Isabelle, to create Isabelle emulator for Mizar where we can reverify translated MML proof scripts. An important sub-goal of this project is to preserve human readability of obtained translation. This sub-goal is often neglected in a machine translation where the result is analyzed only by a computer. Omission of this aspect in the translation certainly destroys the effects of "caring for" proofs readability. The Mizar project from its beginning aimed to make system for the readable formalization of mathematics, where the proof style was designed to imitate style occurring in the informal mathematical practice and where the type system tries to express how mathematicians use mathematical objects and how they categorize them. It is important at this point that solutions used in Mizar have been the inspiration to implement the analogical solutions in other systems.

Research to be Carried Out

First, the planned study will include full reconstruction of the Mizar environment, i.e., the language and the meta-logic basing on the Isar inner syntax notation and defining new objects of logic in Isabelle/FOL, respectively. Second, we develop the adaptation method of the Mizar proof skeleton to the possibilities and limitations of the Isabelle system and the Isar language. Third, the methods that allow to use the information about hidden arguments occurring in the Mizar logic object and enabling solving the problem of overloading symbols in the Isabelle emulator for Mizar will be developed. Fourth, we focus on the existing ways of translation of the information collected in the environment of the Mizar proof scripts and ways of imitation of build-in mechanisms of the Mizar CHECKER in Isabelle system. Finally, we create an automatic translator that translates the proof scripts collected in the MML to Isabelle system. These empirical studies are necessary to obtain the full picture of applicability of the chosen methods that try to emulate the Mizar components and indicate how we can improve these methods.