

DESCRIPTION FOR THE GENERAL PUBLIC (IN ENGLISH)

The project aims at study of *homogeneity* and *minimality* in compact spaces. The study is to be undertaken from the topological and dynamical point of view. In the broadest sense, *homogeneity* is a notion that describes a phenomenon in which a space, population or object is uniform in its nature, composition, or character, without irregularities. Formalizing this notion in topology, one requires from a homogeneous compact space that for any two points there is a one-to-one and onto continuous mapping that can send any given point to any other selected point of the space. Many objects we know fail to be homogeneous and it would seem not difficult to classify homogeneous compact spaces. However, even though it is one of the central questions in topology to classify all such spaces, a potential characterization is known to be very far from immediate, despite the many efforts that were given this subject, as documented by many publications. An inhomogeneity of a space often (although not always) poses some serious obstructions to the way the space can be transformed into itself, forcing dynamical systems on the space to exhibit various kinds of complex dynamics, or at least causing such transformations to always leave a fixed point. Such a behaviour is on the opposite end of the spectrum to a phenomenon that is referred to as *minimality*. Simple examples of minimal systems include restrictions of a dynamical system to a periodic orbit. Minimal dynamical systems are in a sense the simplest ones, and are often called the building blocks for the more complex ones. A minimal dynamical system is the one in which starting from an arbitrary place, under subsequent iterations of the system, we can get arbitrarily close to any other chosen location. Clearly this phenomenon may depend on the structure of the underlying space. An infinite metric space with an isolated point will never admit a minimal system, but rather a collection of "highways" for the system should exist within the space to carry all points almost everywhere and discrepancies within the structure of the space can set obstacles for such "highways" to be present. As with homogeneity in topology, the problem of classifying minimal spaces is a well known and central for the theory of dynamical systems. It is known to be very hard and many papers are published each year on this subject in many important mathematics journals.

Building on the state of the art, the project proposes a parallel approach to homogeneity and minimality. Expanding on his experience with topological spaces that are homogeneous, or almost homogeneous (in various sense), and the study of invariant and attracting sets of dynamical systems, the author will construct new examples of homogeneous spaces, as well as minimal spaces and systems. The author plans to give a particularly careful look to hereditarily indecomposable compacta, that have already proven to be an important source of homogeneous and minimal spaces, such as pseudo-arcs and pseudo-circles. Hereditarily indecomposable continua constitute an important class of spaces, and appreciated for their many intriguing properties. They are often referred to as "very bad fractals", due to their extremely complex structure and manifestation of certain degree of self-similarity. Important examples are known with pseudo-arcs and pseudo-circles appearing in smooth and complex dynamics. Besides topology and dynamical systems, more recently these spaces surfaced also in such areas of mathematics as logic (so-called projective Fraïssé limits) and functional analysis (Conjecture of Wood). However, an effective study of them often requires a high degree of specialization, so they do not seem immediately accessible for study to a general population of researchers, even just in topology or dynamical systems. Given his publication record, the author will not only to discover new important examples dictated by the topic of this project, but also make the entire class of hereditarily indecomposable compacta more accessible to other researchers in topology, dynamics, and other fields of mathematics. Part of the research to be undertaken will search for Hausdorff, non-metric analogues of the known homogeneous 1-dimensional continua. Another part will consider metric "pseudofications" of certain minimal sets from surface dynamics. This should lead the author to uncover new connections to other spaces, not necessarily hereditarily indecomposable, as well as dynamical systems with more complex, or even chaotic dynamics.