One of the deepest principles in mathematics, going back to Descartes' *La Géométrie* in 1637, is the equivalence between geometry and algebra. Geometric objects can be studied by algebraic tools, and algebra can be studied by thinking about it geometrically. Noncommutative geometry is the study of geometry when the associated algebra is noncommutative. Far from being just a generalization of conventional geometry for its own sake, noncommutative geometry is in fact forced on us by the basic principles of quantum mechanics, which require physically observable quantities to correspond to noncommuting operators on a Hilbert space. There are several subareas of noncommutative geometry, each with its own methods. This project is focused on the homological backbone of noncommutative geometry: cyclic homology.

Study of spaces and structures built on them (like bundles of vector spaces, measures of distances) is one of the main aims of contemporary mathematics and mathematical physics. Classification of structures is usually achieved by means of computing invariants: they are designed to distinguish the inequivalent classes of structures. They are indispensable but hard to compute. For this reason, new methods and new techniques are desired and required. One of the main lines of approach is based on symmetries, which were always fundamental in solving problems both in mathematics and in physics. In the studies of quantum spaces, new types of symmetries arise and serve as a crucial organizing principle allowing one to handle highly complicated situations. Such considerations lead Connes and Moscovici to the discovery of a new and unexpected type of cyclic homology known as Hopf-cyclic homology.

Due to its rich algebraic and categorical structure, Hopf-cyclic homology with coefficients has quickly become a new branch of research in itself. The appearance of cyclic homology with general coefficients was desired since the very inception of cyclic theory more-or-less 30 years ago. The periodic version of cyclic homology is a noncommutative generalisation of de Rham cohomology, but operators naturally occurring in differential geometry need a de Rham complex "twisted" by tensoring with the module of sections of a non-trivial flat vector bundle. Therefore, the lack of coefficients capable of playing the role of such a vector bundle was a serious drawback of cyclic theory. It is hoped that experience gained by studying Hopf-cyclic homology with coefficients will lead to solving this fundamental and long-standing noncommutative-geometric problem.

Hopf algebras form the algebraic backbone of quantum groups and are crucial in studying Galois-type extensions of noncommutative algebras. Hopf algebras appear as quantum Galois groups for some field extensions and as quantum structure groups for depth 2 inclusions of factors. The theory of Hopf-Galois extensions provides a unifying algebraic setting for the classical Galois theory and the concept of a principal fibration. Then we can think of such an extension of the algebra of coaction-invariants to its defining comodule algebra as a functor from the category of finite-dimensional corepresentations of the coacting Hopf algebra to the category of finitely generated projective modules. Since this is precisely a key feature of classical principal bundles, we call such coactions principal. In the spirit of classical differential geometry, we study principal coactions through the K-theory of associated modules analysed with the help of the Chern-Galois character and cyclic homology.

Cyclic homology and cohomology is among the most significant discoveries of modern mathematics. Pioneered by Connes and discovered independently by Tsygan, then systematized and turned into a theory through fundamental works of Cuntz, Loday, Quillen and Wodzicki, cyclic homology enjoys a plethora of incarnations and applications throughout a very wide spectrum of mathematics. Among new and promising applications are computations using the celebrated concept of path algebras.

The main research goal of this project is to develop a very general but concrete model of cyclic homology. Our new cyclic model should work for non-unital algebras, have multiplier Hopf algebras governing the symmetry, and enjoy topologically complete coefficients. Achieving our goals would constitute significant advances in one of the most vibrant and widely applicable modern mathematical theory.