Large scale geometry (or coarse geometry) studies objects, usually infinite, seen from the distance. From this perspective a disc looks like a point. Moreover, the natural numbers $0,1,2,3,\ldots$ on the number line seem to lie very close to each other and blur into the half-line of positive numbers.

Mathematicians study different notions of numbers. The most rigid are called fields, e.g. all the real numbers form a field – they can be added, subtracted, multiplied and divided. Integers, that is, $\ldots -2,-1,0,1,2,\ldots$ can be added, subtracted and multiplied, but not always divided, for example 5 is not divisible by 2. Other examples of this type, discussed during any mathematics university course, are matrices. Structures like these are called rings.

Points or vectors on the plane can be added and subtracted, but we can not multiply or divide them. (I refer those of the readers, who have heard about the complex numbers, to points in the 5-dimensional space instead.) Such a structure is called a **group**. Groups are ubiquitous in mathematics – every ring or field contains two groups, the additive group and the group of invertible elements. In particular, invertible matrices form a very important group. Also rotations and symmetries of the plane, or even the 3-dimensional space form a group, which, by the way, can be described as a subgroup of the matrix group.

Extremely important objects in mathematics and its applications in economy or physics are **manifolds**, that is spaces that in close-up look like the plane (or a higher dimensional Euclidean space). The simplest object of this type is the sphere (the surface of a ball), which in close-up reminds the plane so much that the mankind needed some time to realise the Earth is not flat. A slightly more complicated example is a doughnut.

A basic invariant used in the study of manifolds is the **fundamental group** – a group consisting of loops being drawn on the manifold. Such loops, if they contact, can be added – we will obtain something looking like the figure 8, but it is just a bit more complicated type of a loop.

It turns out that such a group can be described in a purely combinatorial way with a, usually infinite, **graph**, that is a set of vertices (points) connected by edges (lines). Each vertex of the graph represents a group element; i.e., a loop on the manifold. We join two vertices by an edge if one of the loops can be obtained from the other by drawing in one of a few basic loops.

For example loops drawn on a ring (or a circle) can be identified with the integers, that is, a loop travelling clockwise around the circle twice corresponds to number 2 and a loop travelling anti-clockwise around the circle seven times corresponds to -7. In this graph two consecutive numbers are always joined by an edge, because when a loop travelling around the circle 5 times is extended by a loop travelling around once, we get a loop making 6 turns.

The graphs of fundamental groups described above (called Cayley graphs) being combinatorial objects are easier to study than manifolds, from which they come. One of the ways of studying these graphs is the large scale geometry. Surprisingly, while looking at the graph (from the distance), it enables us to derive deep conclusions about the manifold (defined locally, as objects looking like the plane).

It turns out, however, that looking from the distance on the graph of a group, we can miss some important information. A solution is to study (still looking from the distance) a sequence of finite graphs, that get bigger and bigger and resemble the initial one more and more – such a sequence is called a **box space**.

Applicability of the theory of box spaces is limited, though, since we are not always able to find appropriate finite graphs approximating the infinite graph. In this situation, we can resort to a similar but more sophisticated construction, the **warped cone** construction. Such a cone, as opposed to cones from mathematics lessons, has no base, but stretches towards infinity.

Cones, including the warped ones, expand as one moves away from the apex. If we could move infinitely far away, we would see loads of Cayley graphs of the group from which the cone origins and sometimes also loads of boxes. The study of such cones and their relations with groups from which they origin is the subject of this research project.

One considered problem is the question whether a warped cone can be put into an infinite dimensional space, called the Hilbert space, without being distorted too much (where the degree of distortion is assessed from the distance, neglecting local deformations) and what are the consequences for the group, from which the cone comes and for the initial manifold. We also think what is the relation between cones not satisfying this condition and reliable telecommunications networks.