"MARTINGALE AND CONCENTRATION INEQUALITIES" – DESCRIPTION FOR THE GENERAL PUBLIC

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In mathematics we are frequently not able to compute some expressions explicitly or the resulting formulas are to complicated to be useful. However, it is often sufficient to estimate the given expression and find other comparable quantities.

The main goal of this research project is the study of two important types of estimates in probability theory: martingales inequalities (and their applications in analysis) and concentration inequalities.

Martingale inequalities. Speaking very generally, martingales are some special sequences of random variables. The study of martingales inequalities most often relies on a method introduced by D.L. Burkholder, which reduces the problem to finding functions (of two or more variables) satisfying certain conditions (implied by the specific structure of the inequality). The construction of such functions can be a very challenging task, but allows us to obtain sharp inequalities with best possible constants. The motivation for proving such inequalities comes from the fact that they can be applied to the study of important objects appearing in analysis. An example of such object is the Beurling-Ahlfors operator, which is an important tool in the study of quasiconformal mappings (i.e. transformations of the plane which "almost" preserve angles) and partial differential equations, and which is connected to a famous open problem posed by T. Iwaniec.

In the project we will study martingale inequalities which lead to new estimates inter alia for the Beurling-Ahlfors operator and consequently give us a better understanding of the behavior of this operator.

Concentration inequalities. The classical isoperimetric inequality on the sphere implies the following phenomenon: on a high dimensional sphere $S^{n-1} \subset \mathbb{R}^n$ the normalized rotation invariant surface measure is concentrated around the equator (or even: around each great circle (!)). Consequently, Lipschitz functions (that is functions with small local oscillations) are essentially constant from the point of view of this measure. This elementary yet highly non-trivial observation set an important research theme in high dimensional probability theory with many applications in various areas of mathematics (e.g. in statistics and the construction of geometrical objects with extremal properties).

In the project we will study the interplay between various approaches which lead to concentration inequalities. We will be especially interested in the connection between those approaches and the class of functions for which we want to obtain a concentration inequality – in the above example with the surface measure on the sphere we introduced the class of Lipschitz functions, but one can also explore situations when the class of functions is bigger (which must pose additional constraints on the measure) or smaller (which in turn allow us to derive results for less regular measures).