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Countless real life problems can be formulated as abstract optimization problems, and many of them are most naturally phrased in terms of graphs. Graphs are fundamental mathematical structures which can be used to represent any kind of objects which are pairwise related. Formally, a graph consists of a set of vertices and a set of edges connecting the vertices. For instance, a telecommunication network may be modelled by a graph in which terminals are represented by vertices and transmission links between the terminals by edges of the graph. Alternatively, in a road network vertices correspond to cities and edges correspond to highways connecting them. Graphs find important applications in many different areas. In computer science they are used to represent networks of communication or transportation, computational devices, flows of computation and many more. Graph theory is also applied to study molecules in chemistry and physics or in DNA representations in biology.

Once a graph theoretical formulation of a real world situation has been established, it is a challenging task to efficiently solve various optimization problems or to extract information from ever growing amounts of structured data. An example of such an optimization problem in a communication network is to place a minimum number of communication antennas to cover all important locations with wireless network. In its graph theoretic formulation the problem corresponds to the problem of finding a small set of vertices in the network graph which dominate all other vertices, that is, which are connected to all other vertices by an edge. In theoretical computer science we call this problem Minimum Dominating Set.

It has long been realized that a large class of important algorithmic graph problems seem to evade all attempts of solving them efficiently in general. The meaning of efficiency and tractability varies, but the Minimum Dominating Set problem, mentioned above, in general is considered intractable with respect to basically all of them. However, the situation changes on restricted graph classes; that is, when we have some a priori knowledge about the properties of the graph modelling a real-life scenario. Namely, instances of graphs arising in applications often have more structure than general graphs. For instance, road or railway maps can be drawn on the plane without crossings (formally, they are *planar* or near-planar) and telecommunication networks are modelled by sparse graphs, that is, by graphs with a moderate number of edges. It appears that on such graphs, the Minimum Dominating Set problem can be solved much more efficiently, also with respect to different notions of tractability.

Researchers have studied many structural properties of graphs which can be used to design efficient algorithms for otherwise hard problems. Among the most important ones are properties of planar graphs or, much more generally, properties of graph classes that exclude a fixed minor. Over a period of more than twenty years, Robertson and Seymour developed a celebrated structure theory of graphs with excluded minors which had an immense influence on the design of efficient algorithms. The general take-away message of this research is that the *sparsity* of a graph can be most often exploited to design more efficient algorithms, however it is only recently that researchers started to investigate this phenomenon more systematically.

Very recent results indicate that a more fundamental property leads to very general algorithmic tractability results. Nešetřil and Ossona de Mendez introduced a natural and very general model of uniform sparseness in graphs, called *nowhere denseness.* All familiar classes of sparse graphs such as planar graphs, bounded degree graphs and classes that exclude minors or topological minors are nowhere dense. In their search for the most general graph classes that have good algorithmic properties, researchers have discovered a rich combinatorial theory of nowhere dense classes of graphs. Using a *logic based approach*, in a collaboration with Grohe and Kreutzer, the applying researcher showed that a wide range of algorithmic problems can be solved efficiently on nowhere dense graph classes. Moreover, it appears that the concept of nowhere denseness can be used to develop elegant general-usage algorithmic techniques for sparse graphs, which can be used to design efficient and strikingly simple algorithms for a wide range of problems in many different areas. On the other hand, it is often the case that nowhere denseness is provably the ultimate limit of where sparsity-based techniques can be employed. However, the theory of nowhere denseness is still very young and largely unexplored; we expect that many beautiful theorems and powerful techniques are yet to be discovered in this direction.

The scientific goals of this proposal are to *apply the newly developed methods for nowhere dense graph classes to efficiently solve specific computational problems arising in different areas of computer science.* In a multi-disciplinary approach we are going to study computationally hard problems from the view of parameterized algorithms, approximation algorithms and distributed algorithms. In each of these areas, we hope to show how sparsity-based techniques can be used to prove new, powerful results and design more efficient algorithms. However, our proposed research diverges from the classic view of algorithmics as a race for the fastest possible algorithms. It is rather a *search for new, very general algorithmic techniques and a quest for understanding their limitations*. In particular, we want to extend the logic based approach to develop a general and broad algorithmic graph structure theory for nowhere dense classes of graphs and beyond, and to understand the most general graph classes on which the logic based approach may work.