# DESCRIPTION FOR THE GENERAL PUBLIC

The source of success of the contemporary physics is the ability to link a physical process to its mathematical model. This is done on two levels, the first of which is a mathematical description of the laws of nature (equations of Newton, Maxwell, Einstein, Schrödinger), and the second are mathematical skills of processing equations thus obtained. The conceptual scheme of physical theories is "matched"to the current development of mathematics. Not only it utilizes the current state of mathematical research (or even is inspired by), but it also provides strong stimulation in development of mathematics in desired directions. It turns out that often some simple, but fundamental for the physical theory, models (like the two-body system, harmonic oscillator at both classical and quantum levels, the Schwarzschild model or the hydrogen atom model) can be analyzed in much detail, which accounts for the success of the theory.

Unfortunately, only a small number of models has this interesting solvability property, which in historical context of models described by differential equations models is often referred to as *complete integrability*. The main cause of failure in obtaining a solution (integration) of a differential equation is usually the non-linearity of this equation. Most phenomena in nature is non-linear, which at the turn of the XIX and XX century led to a pessimistic view that in principle very little can be done in this area, and the reason is not our ignorance but the structure of the problem. Attempts to obtain solutions of non-linear equations and detailed analysis gave way to qualitative techniques and approximate methods.

Therefore, the surprise was discovery in the late sixties of the XX century techniques which allowed for analysis and construction of exact solutions to some specific partial differential equations, of which the most famous are the Korteweg—de Vries equation describing waves in shallow channels, the non-linear Schrödinger equation derived in the context of fiber-optic transmission, or the Kadomtsev—Petviashvili equation in plasma physics. Within the next 50 years of intensive efforts many properties of completely integrable equations have been discovered, and their conection with various fields of pure mathematics, such as algebraic and differential geometry, theory of Lie groups and Lie algebras, spectral analysis of operators has been established. These studies allowed also to find a link between such equations and certain models, investigated independently and by separate techniques, in quantum mechanics and statistical physics. Such unified description of the complete integrability brought out to light the fundamental role of discrete (difference) integrable equations and the theory of Hopf algebras.

During the last decade, we have seen increasing interaction between the theory of discrete integrable equations and combinatorics. Such connection can be inferred from the study of completely solvable statistical physics models, but an important role was also played by the random matrix theory and its close relationship with the representation theory. The research conducted over last 20 years, by the head of the proposed project, on the geometrical description of integrable discrete systems has led to a new and simplistic view on the nature of integrability, as encoded in the incidence geometry statements. The new project is planned to continue these studies with an emphasis on the relationship between projective geometry over the skew fields and quantum integrable models. In addition, it is planned to exploit techniques of formal series to obtain solutions of integrable non-commutative discrete systems and to examine applicability of such solutions to the theory of formal languages and combinatorial Hopf algebras. The other part of the project are applications of the theory of integrable systems to construction and analysis of particular models of theoretical physics relevance or describing nonlinear waves.