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The project is devoted to theoretical analysis of problems in Geometric Optics and Free Material Design. There are subtle mathematical links between these two fields, justifying putting them under the same project.

Geometric Optics studies the propagation of light having small wavelength compared to the dimensions of all the apertures traversed. Light is treated as waves traveling along rays which may be reflected by a mirror, or refracted by a lens. We are interested in the design of surfaces that illuminate a given target that is either close to the source (near field) or at infinity (far field). These designs belong to the engineering branch of nonimaging optics since we are not interested in the image formation but rather in the light distribution. Radiations can be monochromatic (one color), or polychromatic (two or more colors). In the latter case, the main obstacle is the dispersion of light causing aberration in the system. We will study models belonging to either case. These systems are designed in general with numerical software that use ray tracing. The novelty in this area is its connections with mathematical research in nonlinear partial differential equations, differential geometry, geometric measure theory, and calculus of variations. These connections will permit us to solve optical questions in a more precise and general way, and provide a theoretical foundation for their solutions.

To be more specific, we will study three optical models. The first one consists of constructing a near field mirror reflecting rays emitted from a point source; in the second one, we will design a far field lens scattering lights emitted from a planar source; and in the last model, we will study the question of existence of a far field bi-chromatic lens. These surfaces solve a partial differential equation, generally of Monge-Ampère type. We will analyze carefully these PDEs to obtain geometric and analytic properties of the models in study. We will be interested in questions as existence (i.e. theoretical and physical possibility of such designs), uniqueness, regularity and stability. We mention that these systems have various industrial application such as antenna design, street light illumination, telescopes, and freeform optics.

The other part of the project is devoted to the engineering of Free Material Design (FMD), where the goal is to find the optimal shape of an object within the design region. By optimal shape, we mean such a distribution of material, that the object will withstand the prescribed loads. Interestingly, the resulting shape may have voids where there is no material, or the material may concentrate on lower dimensional structures as in the case of trusses. The same framework is used to study the image conductivity problem. Loads in this case are electric voltage applied on a part of the boundary, and the objective is to reconstruct high quality images of body tissues using internal current intensity measurements. These systems appear frequently in biomedicine and tomography.

In the technical language, the problem requires studying functions of least gradient with imposed boundary data. For the sake of convenience, the systems in the literature were considered with loads prescribed at every point of the boundary; and solutions to the least gradient problems are well-studied provided that the data is given on the whole boundary, and the design region is strictly convex. We will relax both assumptions. In reality, the loads are given on a part of the boundary of the design region, the rest is free. Moreover, strict convexity assumption seems quite artificial for practical use, and more general sets will be considered in this project.

Our analysis will help engineering sciences to understand better the design process of new buildings and machine parts. It will also provide additional insights to improve the existent numerical algorithm used in image reconstruction.