DESCRIPTION FOR THE GENERAL PUBLIC (IN ENGLISH)

This research project concerns problems connected with maximal operators and related topics associated with various contexts of orthogonal expansions. In the very classical real harmonic analysis on the Euclidean spaces, the most fundamental operator is the Hardy-Littlewood maximal operator. In this project we plan to consider its counterparts in the contexts of orthogonal expansions. The importance of this object comes from the fact that typically it dominates other fundamental harmonic analysis operators, hence it allows to transfer its mapping properties to other operators. Moreover, maximal operators play a crucial role in solving various problems connected with studying pointwise (almost everywhere) convergence. The most famous application in this direction is obtaining, as an easy corollary to the so-called weak type $(1, 1)$ estimate for the Hardy-Littlewood maximal operator, the Lebesgue differentiation theorem, which says that

$$\lim_{r \to 0^+} \frac{1}{|B(x,r)|} \int_{B(x,r)} f = f(x), \qquad \text{a.a. } x \in \mathbb{R}^d,$$

for all (locally) integrable functions f on $\mathbb{R}^d$; here $|B(x, r)|$ stands for the volume of the ball centered at x and of radius r.

The principal aim of this project is to study maximal operators in several contexts of orthogonal expansions. Actually, we plan to consider maximal operators based on heat and Poisson semigroups which differs a little bit from the one described above. (Note that the classical heat semigroup is an important object which describes propagation of heat in various media.) However, the motivation for studying them is somehow similar. The main question we are interested in is their behavior on basic function spaces, such as L^p, $1 \leq p < \infty$, spaces. The research will be conducted in several frameworks of orthogonal expansions, for instance:

- expansions into orthogonal polynomials in the multi-dimensional unit ball,
- expansions into Laguerre polynomials,
- expansions into Jacobi polynomials,
- Fourier-Bessel expansions,
- continuous expansions of Bessel type.

The results emerging from this project would be new, original and would constitute a noticeable contribution to real harmonic analysis, especially to the area connected with maximal operators and related topics. They would give answers to several important questions that arose recently in connection with the dynamic development of this flow of real harmonic analysis. The research proposed in this project would also require new ideas and essentially new techniques that would be of interest in their own right because of their potential applications in a future research.