

The project concerns the study of Banach spaces of analytic functions on various domains and two classes of operators acting on these spaces - composition operators and inclusion operators. The history of study of Hardy spaces on the unit disk has its roots in the beginning of XX century and it is connected with such mathematicians as G. H. Hardy, F. Riesz or J. E. Littlewood. Over the last decades the study on Hardy spaces developed in many directions becoming a great impetus for the enormous progress in functional, harmonic and complex analysis - Hardy spaces appears in the study of boundary problems for elliptic differential equations, space H^2 is of special significance in the theory of invariant subspaces and many other problems. As time goes it appeared more general objects - Hardy–Orlicz, Hardy–Lorentz and so-called $HX(\mathbb{D})$ spaces, where X is a symmetric space. Moreover Hardy type spaces on general domains (not only simply connected) attracted much attention.

One of the most important area of theory of Hardy spaces is studying special classes of operators. Undoubtedly important role play here composition operators. Composition operators are fundamental objects of study in analysis that arise naturally in many situations. A classical result due to Forelli states that all surjective isometries of the Hardy space $H^p(\mathbb{D})$, $1 < p < \infty$, $p \neq 2$ are weighted composition operators. The problem of relating operator theoretic properties (e.g., boundedness, compactness, weak compactness, order boundedness, spectral properties) of composition operators C_φ to function theoretic properties of generating function (symbol of C_φ) has been a subject of great interest for quite some time. First important result on composition operators was *Littlewood Subordination Principle* which implies boundedness of these operators. Later mathematicians considered more complicated problems for example the compactness of composition operators. This problem after many years was solved by J. H. Shapiro and B. D. MacCluer. Shapiro described compact composition operators in terms of Nevanlinna function, MacCluer using Carleson measures. It should be emphasized that the second of these result (together with the famous Carleson Lemma) led to study another class of operators - inclusion operators $j_\mu: H^p(\mathbb{D}) \rightarrow L^p(\mathbb{D}, \mu)$, where μ is finite Borel measure. This idea is very useful and often used in contemporary research. Note that many interesting properties of composition operators C_φ might be described in geometric properties of symbols φ . For instance if φ maps the unit disk into a polygon inscribed in the unit circle, then C_φ is compact on H^p , $1 \leq p < \infty$. On the other hand if φ is a univalent self-map of the unit disk \mathbb{D} and $\varphi(\mathbb{D})$ contains a disk that is tangent to the unit disk, then C_φ is not compact on H^p , $1 \leq p \leq \infty$.

The study of composition operators links some of the most basic questions you can ask about linear operators with beautiful classical results from analytic-function theory. In the case of multiply connected domains there are also connections with harmonic analysis, so the research topics lie on the border of three major branches of analysis. Our goal is to obtain interesting results in these areas.