

DESCRIPTION FOR THE GENERAL PUBLIC

In our research project, we will study the cohomology of affine algebraic groups in the case of positive characteristic. We plan to compare this cohomology with the discrete cohomology of the corresponding groups of rational points (so-called Chevalley groups). This is a classical line of research in the case of reductive groups, however we plan to focus on the case of non-reductive groups, and this case is not so well-understood. Our aim is to find analogues in the general case of some theorems which are known only for reductive groups. An example of such a theorem is a classical theorem of Cline–van der Kallen–Parshall–Scott (in short, CKPS) about comparison between rational and discrete cohomology and a theorem of van der Kallen–Touzé about finite generation of the ring of rational group cohomology.

It is easy to see that the afore-mentioned theorems are false as stated without the reductivity assumption. Even stating a general version of such a theorem in an adequate way requires usage of a more sophisticated and general notions. Therefore, in our research project we plan to develop the theory of cohomology of groups with endomorphisms (so called difference groups) and to study the connections between algebraic and discrete representations using (among others) the notion of the completion of the derived category. We plan to use these general ideas in a proof of a non-reductive version of CKPS (Conjecture 1) which expresses the discrete cohomology using a completion of the rational cohomology and a version of the theorem about finite generation (Conjecture 3) in the situation of difference rings.

We expect that our research project will improve the understanding of the connections between the cohomology of affine algebraic groups and the cohomology of discrete groups. We also expect that an abstract apparatus introduced in this research project will be useful in studying other problems lying on the border of homological algebra and representation theory.