Popular Science Abstract

"Toric ideals associated to matroids, groups, trees, and other combinatorial problems"

The project concerns problems of algebraic combinatorics – an area of mathematics where interaction of combinatorics and algebra is particularly strong and significant. Our point of view is combinatorial. That is, we plan to apply combinatorial techniques to solve problems that originated from algebra or algebraic geometry. Finally, some of the studied problems will be just loosely linked to algebra, but interesting for its own combinatorial sake.

Toric varieties is a certain, but still quite general class of algebraic varieties. The geometry of a toric variety is fully determined by the combinatorics of its associated fan. When an algebraic variety is constructed using only combinatorial data, one expects to have a combinatorial description of its defining equations. An attempt to achieve this description often leads to surprisingly deep combinatorial questions.

White's conjecture about the toric ideal of a matroid is an example. It asserts that the toric ideal associated to a matroid is generated in degree 2. In particular, that it is generated by quadratic binomials corresponding to symmetric exchanges. White's conjecture resisted numerous attempts since its formulation in 1980.

Closely related to White's conjecture, via graphs on bases of a matroid, is the cyclic ordering conjecture. It provides a necessary and sufficient condition for a matroid to possess a cyclic ordering. As usually in matroid theory one of the implications is easy. The other is an open problem since 1988.

Another family of combinatorially interesting and difficult questions comes from considering toric ideals associated to phylogenetic trees. In particular, we will work on conjectures of Sturmfels and Sullivant that bound the degree in which these ideals are generated.

The aim of the project is to make substantial progress in all the above problems and conjectures.