DESCRIPTION FOR THE GENERAL PUBLIC

Many properties of dynamic systems may be successfully characterised by certain numeric quantities called numeric characteristics or characteristic exponents. They comprise among others: Lyapunov, Perron, Bohl, Izobov, Grobman exponents and generalized spectral radii. These numbers describe different types of stability, trajectory growth rate, or sensitivity of dynamic properties of the system to parametric disturbances. In addition, we may analyse a phenomenon of bifurcation and property of possessing an attractor with the use of these quantities.

During the last decade a considerable interest was observed among specialists in a theory of dynamic models, in which derivatives or differences of fractional orders occur. Such systems in control theory are called fractional order systems. The interest in fractional order systems is driven by a possibility to model them by means of physical phenomena not being able to be modelled by integer order systems.

Historically the first use of the analysis of fractional order to model real phenomena was described in the work in 1823. Abel used the analysis of fractional order to solve a problem of tautochrone. It is based on finding a curve, on which rolling time (without friction) of mass point, under the influence of permanent weight, is the same to its lowest point, regardless of a starting point on this curve. The elegance of this approach encouraged many scientists dealing with the use of mathematics to use derivatives of fractional order in modelling real phenomena. It turned out that the description of phenomena occurring in viscoelastic materials, namely dependence of voltage and stress of materials in time, shape changes under the influence of external factors, such as strength and temperature, with the use of fractional order systems gives better results than in case of the use of total order systems.

A theory of fractional order systems is constantly developing simultaneously with an increase in a number of uses. The increase in a number of uses of fractional order systems implies a constant need to develop the modelling methods, synthesis, analysis and comprehension of dynamics of systems of such kind. Currently at least two journals are edited, which are devoted to differential calculus of fractional order: *Fractional Calculus and Applied Analysis* and *Journal of Fractional Calculus and Applications*. Additionally, many conferences devoted to this theme are held annually. A conference *Non-Integer Order Calculus and its Applications* regularly takes places in Poland. The 2016 edition will be organised in Zakopane from 20th to 21st Sep 2016, and the project applicants are involved in its organisation.

While certain dynamic properties of fractional order systems, such as stability or controllability have been partially tested, then the approach based on characteristic exponents has not been discussed widely in literature. A problem of stability and stabilizability of continuous linear systems of fractional order and the relation between stability and stabilizability are going to be tested in a project based on the introduced numerical characteristics. The final problem, in case of linear stationary systems of integer order, is well-known in control theory, because it is described by the so-called pole placement theorem. This theorem is a basics of many methods of projecting regulators. Searching for an analogue of this theorem for continuous systems of fractional order will be one of the points of the suggested project.

A class of continuous fractional order systems will be studied within the project. We suggest considering the following fractional characteristic exponents: Lyapunov, Perron, Bohl and general ones.
The detailed research objectives are:

1. the description of all fractional Lyapunov, Perron, upper (lower) Bohl exponents corresponding to different initial conditions;
2. the relations between different types of characteristic exponents;
3. the description of dynamic properties of the fractional order system such as stablity, asymptotic stablity, exponential stability, uniform stability, the occurrence of attractor and a bifurcation phenomenon, by means of numeric characteristics;
4. the relations between characteristic exponents of the disturbed system and the non-distrubed system;
5. Millionshikov rotation method for fractional order systems;
6. the study of relations between different types of controllability of fractional order systems and finding necessary and/or sufficient conditions for them;
7. the relations between different types of controllability and different types of stabilizability;
8. the analogue of the pole placement theorem for fractional order systems.