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Orders are ubiquitous in mathematics and theoretical computer science. They represent ways of comparing objects from a set. Orders according to a single criterion are called *linear,* as in a sense they arrange the set into one line—in each pair of objects, one is *less* and the other is *greater*. Orders according to multiple criterions are called partial, as only some pairs of objects can be unequivocally compared in such orders—those in which one object is less and the other is greater according to all criterions simultaneously. If one object is less than another according to the first criterion and greater than the other according to the second criterion, then both criterions together cannot decide which object is less and which is greater, and hence the two objects are *incomparable.*

The reasoning presented above describes only one way of obtaining a partial order. Such an order defined in terms of *d* different criterions is called *d-dimensional*, and the number *d* is called the *dimension* of the order. The primary context of the research proposed in this project is concerned with *computing the dimension* of an order given by an abstract relation comparing objects in a set. Computing the dimension consists in reversing the process presented above—finding, for a given abstract order, the minimum number of criterions that can be used to generate that same order. This is particularly important for practical applications of orders, because a representation in terms of few criterions allows the order to be stored within small memory and allows queries about the relation of particular pairs of objects to be answered quickly.

Order dimension has also big theoretical significance. In some sense, computing the dimension is the analogue of *graph coloring* in the world of partial orders. Graph coloring is a fundamental concept which has been a driving force for the development of graph theory already since the 19th century (vide the *four-color problem*). In a similar way, although to a smaller extent, computing the dimension fosters the development of combinatorics of partially ordered sets. The main objective of the research on order dimension is to understand what properties of the order force its dimension to be bounded and what properties allow it to grow arbitrarily high.

There are three basic goals assumed in the present project. The first one is concerned with the *computational complexity* of the problem of determining the dimension, that is, with the analysis whether exact computation of the dimension of a poset is feasible in practice. The second one is concerned with analysing the extent to which the dimension of an order depends on structural properties of that order. The third one is concerned with so-called *queue* and *stack layouts*, which so far have not been studied in connection to dimension, and which we plan to investigate using similar methods to what we have been using for dimension problems.

The project falls within the line of research which recently has undergone rapid progress that has opened the way to solve known and difficult problems. This progress was a consequence, in particular, of the development of many new proof methods. One of the factors motivating this project is the desire to exploit the potential of these methods as much as possible. We believe that the plethora and attractiveness of problems and methods will allow us to build a strong research group at Jagiellonian University around this project.