Reg. No: 2015/18/M/ST1/00075; Principal Investigator: prof. dr hab. Piotr Andrzej Gwiazda

The aim of the project is to develop mathematical methodsfor hyperbolic systems arising in continuum mechanics and mathematical biology. A fundamental example is the Euler system describing a flow of inviscid fluid. Our attention will be directed also to similar systems, including systems of shallow water type capturing flows of granular media or pressureless Euler equations with pairwise attractive or repulsive interaction forces and non-local alignment forces in velocity appearing in collective behavior patterns.

In numerical analysis we will strongly exploit the particle method, which does not rely on discretization of a state space, what often leads to numerical errors, but on tracking individuals moving along the flow. However a disadvantage of a standard particle method is the limitation to the first order rate of convergence. Within the project we are going to extend this framework in the direction of methods arising mostly from mathematical physics. We mean here e.g. smooth/pseudo particle method together with higher order splitting providing higher order convergence of numerical schemes.

Summarizing, the investigations will be directed into two topics:

1. hyperbolic systems of Euler type
2. particle-based high order numerical methods for hyperbolic systems, including structured population models.

We are going to develop numerous methods, originating from physics (gas dynamics), including optimal transport, particle method, gradient flows, convex integration, measure-valued solutions, relative energy/entropy. Surprisingly, a lot of them may very well be applied in systems arising in mathematical biology, e.g. population dynamics.The studies proposed in the project fit into the trend of the most modern research, the proposed methodology arises from the groundbreaking results of the last decade. Although Poland has a wonderful tradition in classical
functional analysis, but nowadays it is important to follow and to contribute to the latest and most significant achievements. The very recent results of Camillo De Lellis and Laszlo Szekelyhidi have changed the typical approach to partial differential equations, opened the new ways of exploiting abstract functional analytic facts such as Baire category theorem and reached to differential geometry (Nash-Kuiper isometric embedding theorem, results, which were honoured with the Abel Prize for J. Nash and L. Nirenberg). Similarly, the problems of optimal transport are related with C. Villani - the laureate of Fields Medal. The project will be implemented in a collaboration with Jose A. Carrillo (Chair in Applied and Numerical Analysis, Imperial College London).
His expertise comprises long-time asymptotics, qualitative properties and numerical schemes for nonlinear diffusion, hydrodynamic, and kinetic equations in the modelling of collective behaviour of many-body systems such as rarefied gases, granular media, charge particle transport in semiconductors, or cell movement by chemotaxis. We want to benefit from working with our foreign collaborators, who originate from different areas of PDEs, however coupling their expertise brings us to entirely new developments.