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The project concerns estimations of probabilities that random variables depending on a large number of parameters (used e.g. to model complicated random experiments) deviate significantly from their expected values. Results pertaining to such issues have applications in situations when despite a random nature of observed phenomena, one can with high accuracy predict some of their outcomes. A classical example, lying at the foundations of probability theory, is given by the laws of large numbers, according to which an average outcome of many independent runs of an experiment (e.g. a measurement of a physical quantity biased by random errors) is very close to the expected outcome of a single experiment (the true value of the quantity being measured). The laws of large numbers are statements of asymptotic nature, describing the situation with the number of runs tending to infinity, while in many applications it is crucial to derive precise estimations of probabilities of deviations from the expected value for a fixed number of runs. This problem has been investigated already at an early stage of the development of the probability theory, in the first half of the twentieth century many inequalities have been obtained, showing that the probability of observing a significant discrepancy between the averaged outcome and the expected one decays exponentially with the number of repetitions of the experiment. Contemporary theory of probability allows to generalize those results to more complex random variables, depending in a regular way on a large number of parameters. It turns out that situations when despite a complicated random structure, essential numerical characteristics are `almost' deterministic (i.e. the probabilities that they deviate from their expectations decay very fast) appear commonly both in theory and in practical applications. This phenomenon has been first described in an abstract, modern form in the 1970s by V. Milman, who gave it the name of concentration of measure. Since then it has become a key part of the theory of probability, closely related to many applications. It allows e.g. for carrying out numerical computations with randomized Monte Carlo type algorithms, predicting the macroscopic behaviour of physical systems consisting of large numbers of particles, reducing the time of medical measurements or fast processing of large amount of data. The theory of concentration of measure has also multiple applications in theoretical mathematics, not only in probability, but also in geometry or combinatorics, where it is used as a tool for proving existence of objects with extremal properties. Concentration inequalities are also intimately related to isoperimetric problems. The most classical and simplest instance of such a problem is the commonly known fact that among all closed curves in the plane, of fixed circumference the circle encloses the largest area. Isoperimetric results related to concentration of measure are generalizations of this observation to more complicated, high dimensional situations and appropriate counterparts of the notions of circumference and area.

In all the aforementioned applications it is crucial to obtain precise estimates on probabilities of atypical situations. Results of this kind are known usually only for those numerical characteristics which depend in a very regular way on the parameters (in most theorems one assumes that the corresponding functions satisfy a property called the Lipschitz condition). Moreover, known results concern mostly very special types of random processes, with many restrictive conditions, limiting their applicability.

The goal of the project is obtaining optimal concentration inequalities in more general situations, either by relaxing the assumptions concerning the investigated functions or by extending the class of studied processes. In particular one will analyse functions which do not satisfy the Lipschitz condition. One will also study convex functions of general random variables (with many applications in asymptotic geometric analysis, examining typical behaviour of high dimensional convex bodies), as well as models of random permutations, motivated by problems of mathematical biology and physics. Inequalities for convex functions will be investigated under much milder assumptions than those used at present. One of the objectives of the project is to describe the interplay between various mathematical descriptions of the concentration of measure phenomenon for convex functions, parallel to analogous results in the classical theory for Lipschitz functions. One will also study inequalities for sums of random variables in situations more general than encompassed by existing theory, which will lead to new applications in the analysis of randomized algorithms and in statistics (e.g. in statistical inference for diffusions and in machine learning).

The project is related to many areas of mathematics. Beyond the theory of probability, essential parts will be played by functional analysis, geometry, the theory of partial differential equations (in particular Hamilton-Jacobi equations, closely related to mechanics) or the theory of optimal transportation.