The main goal of the project is to make progress in several hard problems of Ramsey Theory -- a wide area of discrete mathematics studying unavoidable regularities in partitions of combinatorial structures. Most of them could be described via coloring of the integers avoiding certain patterns on arithmetic progressions with specified gaps. An archetypical result providing our main motivation is the famous theorem of Van der Waerden asserting that in any finite coloring of the integers one always finds monochromatic arithmetic progressions of any finite length.

We plan to attack two important conjectures concerning variants of the problem in which steps of arithmetic progressions are restricted to a fixed subset of positive integers: the Katznelson-Ruzsa conjecture (on the chromatic number of integer distance graphs) and the Brown-Graham-Landman conjecture (concerning so called large sets of integers).

Further inspiration comes from another famous result -- a theorem of Thue asserting the existence of arbitrarily long nonrepetitive sequences (no two adjacent blocks are identical) over finite alphabets. We formulate a few new problems relating arithmetic progressions and nonrepetitive sequences, including one with geometric flavor, inspired by the notoriously difficult Hadwiger-Nelson problem on the chromatic number of the plane (the least number of colors needed for coloring of the plane with no two points at distance one and same color).

Two other problems we wish to attack are of more number theoretic nature. One of them is a natural variant of the Lonely Runner Conjecture stating that if n runners are running with different speeds around a unit circle, then for each of them there is a moment in time when there is no other runner within a distance less than 1/n. The other one is a new intriguing conjecture connected to the celebrated Graham's greatest common divisor problem: among any n positive integers there are at least two with arithmetic distance at least n). Both of these classical problems can be formulated using combinatorial properties of so called central arithmetic progressions.