The general objective of the project is to develop complete theoretical formulation and effective finite element method (FEM) model of contact between higher order solids.

Finite element method is generally accepted method of solving system of partial differential equations by a division of domain (physical model of a solid) into subdomains (elements) with distinguished points named nodes. At the level of every element unknowns are interpolated with interpolation functions which values are determined in the nodes of element. The main idea of FEM is to transform a system of partial differential equations into a system of algebraic equations, which unknowns (usually displacements) are nodal degrees of freedom. Hence its universality FEM is used for a large class of physical problems including solid mechanics which is subject of this research project.

Higher order solids are defined as materials whose response depends not only on the first order derivatives of displacement, but also on derivatives of higher order resulting in non-local response of solids. It means that response of material in given point is dependent not only on state variables in that particular point but also on values in its surrounding. Example of such solids are: second order elasticity, where the response of a material depends on second order derivatives of displacement and gradient-enhanced crystal plasticity which response depends on serious defects in crystalographic lattice (dislocations). This non-local effect in specimens with size of centimeters and bigger is not significant, however, when size of the specimen decreases to micro- or nanometers the effect should not be neglected. In the purely displacement formulation of FEM the effect requires using interpolation functions whose first order derivatives are continous on the inter-elements boundaries, in other words field of the first order derivatives of displacement has to be continous in whole domain. There are elements that satisfy this requirement, but in classical FEM their efficiency is not satisfactory. Another method to circumvent this inconvenience is to adopt independent interpolation schemes for displacement and for unknowns being a function of the first order derivatives of displacement and to couple these fields to each other. In that way classical interpolation polynomials can be adopted. Three main methods for coupling mentioned above fields are used: penalty formulation, Lagrange multipliers and Helmholtz partial differential equation. Both higher order solids and adopted interpolation schemes result in another feature not present in classical materials, which is new boundary conditions. Together with the aforementioned methods, several combinations of interpolation orders for displacement and higher order unknowns will be implemented in FEM software in order to establish reliable, efficient finite elements in terms of accuracy, convergence rate and numerical cost. In particular, suitability of the specific formulations for contact analysis will be thoroughly investigated.

In classical contact formulation of two deformable solids (for the sake of brevity let consider frictionless contact) for given boundary conditions (forces and displacement) displacement field and interaction forces (reactions, contact pressure) have to be evaluated under two fundamental conditions: solids can not penetrate each other and separation after contact is possible. For satisfying these conditions, which may be written as inequality constraints, special finite elements named contact elements are used. This problem for classic ("the first order") materials was thoroughly investigated by many researchers, however, for higher order solids, defined shortly in preceding paragraph, there is no contact formulation which impose constraints on higher order unknowns as a function of contact pressure.

The major aim of the project is to develop a continuum formulation and the corresponding two- and three-dimensional contact elements allowing, apart from the classical constraints enforced on the displacements, to constrain higher order unknowns as a function of contact pressure. To the knowledge of the project investigator there is no such model available in the literature.

The final objective is to apply developed numerical models to contact problems of higher order solids and analyse size effects observed in experimental tests with proposed numerical framework. Results obtained from simulations will be compared with existing experimental data and analytical solutions available in the literature.

The research project will be realised using advanced tools for generation and optimization of finite element codes that are available in \emph{AceGen/AceFEM} system, which combines advantages of symbolic algebra available in \emph{Mathematica} system and automatic differentation techniques. This tool allows to develop in an efficient way an arbitrarily complicated finite element in terms of kinematics of the element and constitutive equations governing response of the material. Established numerical model will be used to simulate complex boundary value problems of contact at small length scales for second order elasticity and gradient-enhanced crystal plasticity materials.

Development of the proposed contact formulation which includes imposing constraints on higher order degrees of freedom as a function of contact pressure will improve understanding of micromechanics of contact between higher order solids. Developed theoretical model and its numerical implementation will be ready to use in practical applications and by other researchers to investigate behaviour of higher order solids in contact problems, including influence of boundary conditions on overall response.

In a wider perspective, the established model will lend a new insight into size effects observed in experiments of micro- and nanoindentation tests. Furthermore it may contribute to better understanding of micromechanics of polycrystals at large plastic strains including phenomena occurring at the metal grain boundaries.