The origin of this project was the following interesting theorem proved recently by the head of the project:

Theorem. Let $n \leq m$ and let $\Omega=\Omega_n^m(d_1,...,d_m)$ denote the space of polynomial mappings $f: C^n \to C^m$ of multi-degree bounded by $d_1,...,d_m$. Then there is a Zariski open dense subset U of Ω such that every mappings f, f' from U have the same topological type.

This means that topologically almost all mappings from Ω are the same (in particular every polynomial mapping from Ω is a deformation of mappings with fixed topological type). Hence we can say about topological invariants of a general mapping f from $\Omega_n^m(d_1,...,d_m)$. In particular from topological point of view they have the same singularities, the same critical set, the same discriminant etc. We show that such a general mapping has relatively nice and simple geometry.

The aim of our project is to describe topological invariants of general mappings in an effective way. In particular we compute the number of cusps of a general polynomial mapping f from $\Omega_2^2(d_1,d_2)$, the genus of the singular locus of f, the number of nodes of the discriminant (this work is almost finished now). We do the same for mappings $f: C^2 \to C^3$, $f: C^2 \to C^4$, $f: C^3 \to C^3$ (here we compute the number of swallowtails and we describe the topology of the curve of cusps), $f: C^3 \to C^4$ etc. We would like also to obtain general formulas for mappings $f: C^n \to C^m$, however this seems to be very difficult. Currently we are confident that we will be able to provide substantial results for $f: C^n \to C^n$ mappings for arbitrary n and $f: C^n \to C^m$ mappings for small values of n. We will also introduce a new definition of stability, which is convenient for polynomial mappings.

Topology of smooth mappings $f: R^n \to R^n$ has been studied by many authors. One of the precursors in this field was Whitney, who proved that a general smooth mapping $f: R^2 \to R^2$ has only twofolds and cusps as singularities. Later the authors focused only on research of local nature, like Gaffney and Mond who gave formulas counting the number of cusps of a general perturbation of a finitely determined holomorphic map-germ $f_0: (C^2,0) \to (C^2,0)$. The approach to studying $C^n \to C^m$ polynomial mappings we propose is completely new. In some sense we build the singularity theory for global polynomial mappings and create the suitable tools for doing this. Also the scope of the proposed research is unprecedented. Until now formulas counting singularities of given types were obtained only for local mappings $f: (C^n,0) \to (C^m,0)$ for small values of n and m. The methods we are introducing allow obtaining recursive formulas and providing results for global $C^n \to C^n$ mappings and for arbitrary n.