

The subject of the project lies in between two important areas of research: symplectic geometry and algebraic topology. Symplectic geometry is a section of differential geometry, originating from classical mechanics. The formalism of symplectic geometry generalizes natural concepts in mechanics, such as positions and momenta, and the objects investigated within this theory such as the moment map (the angular momentum analogue) or Hamiltonians originate directly from classical physics.

Both in physics and mathematics, one of the key problems is finding invariants - qualities which remain unchanged under certain modifications of the system in question. For example, in physical system we might be interested in knowing which quantities are preserved in time or remain unchanged under the symmetries of the system. For a system exhibiting the rotational symmetries we are often interested in knowing the quantities which remain unchanged under rotations etc. An important theorem of classical mechanics is the Noether theorem, stating that to every conserved quantity corresponds a certain symmetry of the system (for example, the conservation of energy principle corresponds to translations in time, while conservation of angular momentum corresponds to rotational symmetry).

A mathematical equivalent of the above classical mechanic description are the symplectic spaces (spaces with distinguished position and momentum coordinates), equipped with an action of a group. A group is an algebraic object used to describe transformations which are the symmetries of the investigated space. Studying the invariants of the group actions on symplectic spaces is therefore analogous to investigating the conserved quantities of the physical system.

Algebraic topology is one of the most effective theories applied to the study of topological spaces, which are a natural generalization of geometry, in which the spaces are considered equivalent whenever they differ by a continuous deformation, which is also continuously invertible. Algebraic topology assigns to topological spaces certain algebraic objects (for example their symmetry groups), translating the problem of studying continuous deformations of spaces into investigating maps between algebraic objects, which are quite often easier to handle. The aim of algebraic topology is therefore to assign to a given space some algebraic invariant (invariant under certain class of deformations), and then translating the given problem into purely algebraic terms.

One of the useful invariants used in algebraic topology are cohomology theories. For spaces with a group action (for example the describes above symplectic spaces) a natural cohomology theory is the equivariant cohomology - taking into account not only the topology of a given space, but also the group acting on it. Equivariant cohomology is the main objective of this project. The Gysin homomorphism, a term appearing in the title of the project, is a generalization of the notion of an integral. The formulas I have proven in some special cases suggest that such integrals can be expressed in the language of complex analysis, as residues at infinity of certain complex functions.

The aim of the project is to show how, for a large and important class of space consisting of homogeneous spaces of Lie groups, one can apply both symplectic geometry and equivariant cohomology theory to describe the Gysin homomorphism. Such a description would allow to generalize the partial results on Gysin homomorphism that I have obtained, and endow them with a geometric interpretation, allowing to use methods of symplectic geometry more efficiently to study equivariant cohomology. Additionally, such a description in terms of residues might simplify computations related to the calculus of symmetric functions, whose investigation is often combinatorically involved.