This project concerns the research on certain class of complex algebraic varieties. A *complex algebraic variety* is a geometric object described by polynomial equations. The smoothness is an important property considered for algebraic varieties. A variety is smooth (or nonsingular) if it locally `looks like' a vector space. Not every algebraic variety is smooth. An example of singular algebraic variety is given by cusp, that is the plane curve given by the equation $x^2 = y^3$.

A *resolution of singularities* allows us to modify a singular variety to get a nonsingular one. It can be seen as a `surgery' which cuts out the singular locus and replace it with some subvariety. For varieties of dimension greater than one such operation is not uniquely determined.

We are interested in *quotient singularities*, which are obtained via action of finite groups of vector space. For example the action of -identity matrix on the (x,y)-plane gives the variety that can be identified with the surface in the (x,y,z)-space given by equation $x^2 + y^2 + z^2 = 0$.
And if we consider the action of diagonal matrix with entries i and -i on the diagonal we obtain the variety $x^2 + y^2 + z^4 = 0$. Both varieties are surfaces, which are singular in the point x = y= z = 0.

In the case of surfaces there is one distinguished resolution of singularities which is called the minimal resolution. For the first example presented above it is given by replacing the singular point with~the~projective line P^1 (which is topologically the same as the sphere) and for the second one - with the chain of three such lines. In higher dimensions there is no such distinguished resolution. Instead, for a fixed resolution one may consider a universal algebraic object which allows to reconstruct all resolutions that are small modifications of the fixed one. This algebraic object is called *Cox ring*.

In the recent paper of M. Donten-Bury and J. Wiśniewski considered the action of 32-element group on the four-dimensional complex vector space. It turned out that in this case there exist 81 `minimal' resolutions that are connected by sequences of certain modifications on small subsets (so-called Mukai flops).

The central objects of our research are symplectic quotient singularities. They are important in various branches of mathematics. They correspond to hyperkähler manifolds in the differential geometry, which are the manifolds with Riemannian metric satisfying certain special properties.
In this project we will work on the generalizations of the constructions presented in the paper of M. Donten-Bury and J. Wiśniewski, and developed in the the joint work of PI and M. Donten-Bury. We propose to investigate Cox rings of symplectic resolutions of such quotients, i.e. resolutions that preserve symplectic structure, and Cox rings of Q-factorial terminalizations, which generalize the concept of symplectic resolution. Using the obtained results we are going to study relations between various terminalizations. Apart from the development of the general theory of Cox rings and possible generalizations to other classes of singularities our research is motivated also by direct applications in the hyperkähler geometry. New symplectic resolutions can be used there to produce the examples of compact hyperkähler manifolds via generalized Kummer construction.