

The project aims to study the interplay between two important areas of pure mathematics: operator algebras and dynamical systems. The research will be undertaken from the perspective of the classification programme for  $C^*$ -algebras, which seeks to classify complicated mathematical objects ( $C^*$ -algebras) by less complicated invariants (ordered abelian groups paired with Choquet simplices).

Pure mathematics, by its very nature, is an abstract subject more often driven by the beauty of its constructions and concepts than by immediate practical applications. For this reason, it may seem that the motivation behind research in pure mathematics is simply an intellectual pursuit, often too abstruse for the general public. However, many ideas underlying the various fields of pure mathematics are indeed inspired by the natural world which, although abstracted, are examined by the mathematician who is driven by curiosities familiar across all subjects. In this case, we have two main subjects of study: Operator algebras and dynamical systems, and our curiosity stems from two natural questions: When can we tell distinguish between two complicated objects? How much can two related but different fields say to one another?

Operator algebras stem from the study of quantum mechanics. In quantum mechanics, one must take care about what order observations are made, that is to say, the observables of momentum and position do not commute. If we were to take these two measurements and represent them as a point on an x-y-plane, taking first the x coordinate and then the y could yield a different value than taking first the y and then the x. In mathematics, loosely speaking, the study of 'points in a space' is equivalent to the study of the functions on that space. Functions on a space also commute. We may, however, drop the commutation relation between functions when we consider operators on a (Hilbert) space and this leads to the study of such objects as von Neumann algebras and  $C^*$ -algebras.

Dynamical systems, meanwhile, seeks to build a mathematical framework for the study of the change a system over time, for example, the flow of a river, or for studying symmetries of a space. In the easiest to understand case, one may represent the system as points in a space with an action that moves the points around as time progresses. At given increments, the system is in a particular state. This amounts to an action of the integers on this space. One may generalise this notion to actions of arbitrary (topological) groups acting on a space by homeomorphisms; this can lead to understanding symmetries of that space, for example the cyclic group of six elements can act on a hexagon by rotations, while the cyclic group of eight cannot; this tells us about the rotational symmetry of the hexagon.

Further abstracting, we may apply the noncommutative approach to the notion of a dynamical system by replacing the space with a  $C^*$ -algebra and even the group with a quantum group. In fact, these two subjects--operator algebras and dynamical systems--have a long history of mutual influence. In particular, groups acting on spaces, and more generally, (possibly quantum) groups acting on  $C^*$ -algebras, give rise to  $C^*$ -algebras via the so-called crossed product construction (and other related constructions which can encode the dynamics). By imposing certain regularity properties on the actions, we can control some of the structure of the  $C^*$ -algebras, allowing them to be studied from the point of view of the classification programme for  $C^*$ -algebras.

Classification is a natural methodology for understanding our surroundings. Not only do we see this as a recurring theme across science (taxonomy as the classification of biological organism or the periodic table aligning related chemical elements, for example), but it is indeed a basic human curiosity: this collection of buildings is a town while that is classified as a city; this book is fiction that one is a biography; this group of people is one family, that is another. What sort of information is required to tell these things apart?

In mathematics, classification is an indispensable tool for the proper understanding of mathematical objects, and correspondingly, it has played a central and recurring role in the subject. What properties can we expect to see in our mathematical objects? One may take certain properties for granted, only to find exotic and pathological examples where they do not occur. When do we have an isomorphism? Can we identify particular invariants that allow us to decide when we have an isomorphism without having to rely on a bare hands construction of such a map?

The classification programme for  $C^*$ -algebras seeks to classify (separable nuclear)  $C^*$ -algebras by K-theoretic invariants as well as to examine their structure and regularity properties. This Project will use this classification viewpoint to study the  $C^*$ -algebras related to topological and quantum dynamical systems. In addition, effort will be made to import classification techniques into the realm of dynamical systems. I will focus on structural properties and classification. By nature, it is interdisciplinary: the overarching goal is to contribute cutting edge research to the classification programme by importing techniques from von Neumann algebras and measure theoretic dynamics to the setting of  $C^*$ -algebras and topological and quantum dynamics. I aim to provide new links between these areas in effort to enhance understanding in both subjects.