

Several problems are proposed for research which all belong to the area of dynamical systems theory. The theory historically grew out of celestial mechanics at the end of the 19th century and its founder is considered to be the French mathematician Henri Poincaré. He was the first to prove that problems of celestial mechanics such as the description of motion of three gravitationally interacting bodies cannot be solved in a closed form. A need therefore arose to develop qualitative tools to describe various interesting aspects of motion. A classical question which remains unsolved today is about the stability of the Solar System - whether it should be expected to go on without qualitative change forever, or at some point its structure will become different, for example by ejecting a large planet into the interstellar space. Stability in a broader sense is the question whether a small change in the initial state of the system will remain small under its evolution or can grow at some point in time to become significant.

As it turns out celestial mechanics is just an early example of mathematical modeling of change. By now such models are used in practically every branch of natural and social sciences. They usually cannot be solved in a closed form and most of the time are examined by computer simulation. Such simulation, however, could be meaningless if the system in question lacks stability and initial conditions are not chosen appropriately. Some level of qualitative understanding is therefore indispensable.

Problems selected for research in this project are on the boundary of chaos, which means that stable and unstable motions are mixed together. Compared with models which appear in practical applications, the systems proposed for this research are extremely simple and in many cases described by a single real or complex parameter. This reflects the difficulties of the theory dynamical systems which encounters many challenges even in such greatly simplified cases. Specifically, four research objectives are proposed.

The first objective has to do with iterations of polynomials in the complex plane. A well-known object, even from popular science, is the connectedness locus or Mandelbrot set. Our detailed proposals concern the fine structure of the boundary of this set.

The second objective is about the metric attractors for a certain class of mappings of the circle and complex plane. A metric attractor describes the typical long-term behavior of orbits of the system which are chosen at random. The particular class of systems chosen here is noted for being difficult to study. The detailed plans in this objective include computer-aided study.

The third objective is about the dynamics of a different class of circle mappings with connections to closed orbits which appear in theoretical mechanics.

The fourth objective is related to problems of statistical physics on one hand and applied science such as optimization and image processing on the other. The problem with optimization algorithms is that they may converge to solutions which are only local in the sense that they are optimal under small perturbations but still far globally optimal ones. A type of local solutions will be examined with the goal of setting bounds for their existence and finding ways to avoid them.