Diffusion is one of the most fundamental natural phenomena which is present i nearly all the time- and space-scales. The laws of diffusion govern such processes as random motion of microscopic molecules inside the cells, heat distribution, dispersion of contaminants inside aquifers, population migrations and also galactic dynamics. All those phenomena have completely different origin but their mathematical description is very similar. This description consists of partial differential equations of the parabolic type. In this case, the equation is called "partial" because it binds together the temporal and spatial derivatives which in turn translates into the description of the time and space evolution of the system.

In this project we investigate the water diffusion inside some porous media such as soil or construction materials. The equation that describes this process, i.e. tells us about the evolution of the wetting front, in this case becomes nonlinear. This nonlinearity means that the speed of water wave propagation depends on the moisture concentration level which complicates the mathematical analysis yielding very interesting results. Moreover, as it appeared in the last decade, in some materials (like zeolite or siliceous bricks) the diffusion undergoes a slower dispersion rate than is expected from the main parabolic equation. This phenomenon is called a subdiffusion which belongs to a broader family of diffusive processes (along with superdiffusion). One of the possible explanations of this anomaly is the fact that during imbibition some parcels of water can be trapped in particular regions of the medium. This causes the overall flow to propagate more slowly. As we showed in one of the previous works which constitute the basis for this project, the trapping phenomenon causes the main parabolic equation to become nonlocal. The integer-order derivative, in this case, becomes the so-called fractional derivative.

The realisation of the project will help to investigate the resulting nonlocal and nonlinear subdiffuion equations. We will prove some estimates on the wetting front position and the cumulative moisture intake, which are central for applications. Moreover, we plan to take up some purely theoretical conjectures which proofs could yield many interesting and constructive estimates. In the background, we plan to conduct a thorough numerical analysis with the appropriate analysis of its convergence. All those results will provide very important insights about the phenomena inside the anomalous porous media and will give us an opportunity to investigate some interesting mathematical objects (nonlinear integro-differential equations).

The significance of the project will have an impact not only in purely theoretical considerations. The theorems we will prove will give us some accurate estimates on the wetting front and the speed of water imbinitions. As the experiments show, many important materials enjoy subdiffusive characteristics underlining their importance in the industry (not only in civil engineering). Our analysis will help to understand the behaviour of those materials. The knowledge of the moisture dispersion is crucial for determining such material's properties as its durability, ageing, weathering and the rate of transport of many chemical solutions. Mathematical analysis is necessary to broaden this topic and provide many important consequences.