We hold beliefs of different strengths, which philosophers call “degrees” of the given belief; we will call the collection of propositions a given agent holds an opinion about, together with her degrees of belief in those propositions, the agent's “credal state”. The following are some of the more important questions in the philosophical sub-field this project belongs to, that is, in formal epistemology (which concerns the application of mathematical tools to studying philosophical problems related to human cognition): can anything be said, in general, about the credal state of a rational agent? Are some credal states marred by some disadvantages making them irrational? Moreover, how should a rational agent's credal state evolve with new information? How does the answer to the last question vary with the type of the incoming information?

There is, of course, no single measure of rationality. Epistemologists have long used arguments for epistemic norms governing the features of credal states which appealed to “Dutch Books”, that is, series of books such that each separate book in the series is considered as fair by the agent but all of them taken together lead to her sure loss. A classical result in this field (by Bruno de Finetti) is that we can avoid Dutch Books (i.e. avoiding a certain form of irrationality) is possible if and only if our credal state is a classical probability space, the most basic probability-related structure in mathematics.

Arguments using Dutch Books have some philosophical drawbacks stemming from their pragmatic nature; in recent years epistemologists have attempted to present arguments based on the concept of “epistemic inaccuracy”, appealing to the intuition that a rational agent should take care that her beliefs are as close to truth as possible. There are many options regarding the mathematical modelling of inaccuracy, that “distance from truth”; a popular argumentation scheme for epistemic norms governing e.g. the modification of one's credal state in view of new data is to show a mathematical theorem to the effect that only by proceeding in a specific way the agent will minimise her expected epistemic inaccuracy. The main goal of this project is to create a new approach to modelling epistemic inaccuracy and the appropriate responses of a rational agent to new data; we especially want to apply it to situations, in which there is no consensus in the literature concerning the rational credal state update. The reason for undertaking research on these issues is simply their importance; and also the fact that we already have preliminary results concerning some of them, which suggest that we will be able to complete the proposed research tasks.

We will begin with a scrupulous analysis of the most philosophically developed system focusing on epistemic inaccuracy, proposed by H. Leitgeb and R. Pettigrew. Our preliminary results suggest, that the inaccuracy measure used by these Authors is not, contrary to their claims, sufficiently philosophically motivated, in some situations leading also to unintuitive verdicts. We believe it is advisable, then, to create a competing proposal, probably using a different inaccuracy measure. We well also explore the issue of the rational response by an agent in situations in which the new information boils down to conditional probability (“if A, then there is a 40% chance that B”) or probabilities of overlapping events (e.g. “there is 40% chance of sunny weather, and there is 80% chance that the next bus will be late”): it is not evident, how exactly a rational agent should update her credal state after receiving such data; the current epistemological literature features diverse proposals.

A part of the project concerns topics from philosophy of physics. Researchers in that field use “nonclassical” probability spaces: it is possible that some two events to belong to such a space, but their conjunction (the event corresponding to the joint occurrence of the two) does not; this may be the case e.g. when the events correspond to outcomes of measurements which cannot be performed together. It has recently become popular to associate also nonclassical probability spaces with the degrees of belief of a rational agent. Theorems regarding these issues typically use some variant of the Dutch Book notion, and one of the goals of the current project it to propose an appropriate theorem (with a sense similar to the above mentioned result by de Finetti) based on the notion of epistemic inaccuracy, the definition of which in physically “nonclassical” contexts is not obvious. We have obtained preliminary results connecting the existence of classical extensions of a given nonclassical space with the fact that if the probabilities featuring in that space are interpreted as degrees of belief of some agent, then that agent would not be susceptible to a Dutch Book. This issue, and also the more technical matter of general formal conditions governing the existence of classical extensions of nonclassical spaces, is widely discussed in contemporary philosophy of physics, and is also connected with epistemology.

Tackling the above issues requires also an analysis of the various notions of “extensions of probability spaces” in the literature; such “extensions” are frequently used in modelling e.g. the addition of new factors to previously conducted analysis of some population, increasing the number of factors of interest to the agent whose behaviour we want to model, the income of new information to the agent, or in philosophy of physics, when we want to ask whether the phenomenon we are describing is inherently nonclassical. It turns out that in the literature there are two notions of “extension” of incomparable logical strength: neither is a special case of the other. We want to explore the possibility of creating a more general notion of extension, applicable in different philosophical contexts, and also to analyse the precise scope of application of the notions already in use.