Functioning of living organisms is a result of many complex biochemical processes taking place in cells. On of them is gene expression and regulation that is production of various types of proteins based on genetic code in DNA. Protein molecules interact with themselves, activate or repress their own production. In this way they take part in information transmission which is indispensable for cells to to adapt to changing environment, to make decisions, for example to fight malicious viruses. Evolutionary biology, ecology, and social sciences deal with systems of many interacting individuals, to desribe changes in time of frequencies of various fenotypes, their extinction, emergence, and coexistence. In particular, on evolutionary level, very useful is evolutionary game theory. There payoffs of players using various strategies (fenotypes) are interpreted as numbers of offspring which inherit these strategies. In both cases, in molecular biology in micro scale and in evolutionary biology in macro scale, we have to deal with systems of many interacting entities.

Behaviour of above systems, their time evolution and properties of stationary states, problems and questions posed in biological sciences, are inspiration and motivation for us to construct appropriate models, analysis of which requires development of new mathematical theories and techniques. Mathematical results and their interpretation will allow us to at least partially answer some biological questions.

Usually, time evolution of biological systems is modeled by deterministic differential equations which describe rate of change of concentations of substrats and products of biochemical reactions or frequencies of various fenotypes in evolutionary games in evolutionary, ecological and social processes. However, in many cells number of protein molecules and interacting individuals in evolutionary games is low. Small systems are subject to stochastic fluctuations.To describe them one has to use probabilistic methods.

Moreover, it was usually assumed that reactions in biochemical systems take place instantaneously, and their effects are immediate. In reality, all biochemical processes take a certain time, there is a substantial time delay between the beginning of a reaction and the appearance of new products in the system. Similarly, in ecological and evolutionary models, results of interactions between individuals may appear in the future, and in social models, players may act, that is choose appropriate strategies, on the basis of the information concerning events in the past.

In our project we will investigate effects of time delays and stochastic perturbations on behaviour of biological systems, in particular on the emergence and stability of cycles and properties of stationary states. We will develop approximate methods connected with various time scales of biological processes. From a mathematical point of view, we will develop perturbation theory of stochastic processes and stationary probability distributions of aproppriate random variables.

Specific formulas obtained in the project will be very useful in quantitative biology, to infer various parameters of biochemical reactions and to speed up calculations and numerical simulations of large bio-systems. In evolutionary games, our goal will be to see how the presence of time delays changes standard results concerning stability of stationary states and extinction of strategies.