

One of the main objectives of this project is to investigate properties of solutions to Hamilton equations. Those solutions describe evolution of physical system. In particular Hamiltonian systems apply to classical mechanics or electrodynamics. For a chosen Hamiltonian (energy function) it is often impossible to find an explicit solutions. One can try to use computers to find approximated solutions. However, we prefer to use topological methods. Topological methods still do not give exact solutions but they can give, for example, a lower bound of the number of periodic solutions that Hamiltonian system possesses. An advantage of using topological invariants is that they give same answers for all similar (in the mathematical sense) Hamiltonians. This is important due to the fact that in practice we do not know precise form of Hamiltonian.

The invariant we have chosen to work with was invented by C. Conley - an American mathematician. However, we cannot use his ideas directly. This is because of the fact that the problem we are considering takes place in an infinite dimensional space. In infinite dimensional spaces one cannot trust in the intuition anymore. This is what we would like to do in the second part of our project - work out a reasonable framework for infinite dimensional Conley index. Some construction of such an invariant were already done by the contractors. However, they still need further investigation. In particular, we would like that our invariant is easy to compute which is very important for applications.

Third and the final part of our project concerns with applications to Seiberg-Witten equations. Those equations were first studied by physicists but now they have also great meaning for mathematics, including low dimensional geometry. For many years mathematicians were interested in classifying manifolds. Classification of one and two dimensional manifolds is well known. It turned out that manifolds of dimension equal or greater than five are easier to deal with than those of dimension three and four. Seiberg-Witten equations give us a useful tool to study geometry of three and four dimensional manifolds. In this project we would like to show that one combine both - Seiberg-Witten equations and the Conley index to get a new invariant.