In modeling many phenomena in the surrounding world stochastic processes play a signi cant role. Examples include different processes: physico-chemical (e.g. dffusions), biological (genes transfer between consecutive generations) or economic (modeling price changes of nancial assets). Using the last example, we will explain what the natural questions about stochastic processes are and what speci cally we are going to explore.

In 1900, L. Bachelier proposed modeling of price changes on the stock market by means of a random process, presently known as a Brownian motion B_t. However, since increments of the Brownian motion does not depend on its value at the present time, the better model is such that the increments are proportional to the initial value. Intuitively it is clear that it is much more likely that the price will increase by 10 EUR if the initial share value is 100 EUR than if it is 5 EUR. This revised model was proposed by P. Samuelson in 1963: if S_t denotes the price of a share at the time t then $S_t = se^{\sigma B_t + \mu t}$, where $s>0$, σ and μ are some constants.

Using this model, Black, Scholes and Merton in 1973 obtained a formula, known as the Black-Scholes price formula, describing the valuation of options, for which in 1997, Merton and Scholes received the Nobel Prize in economics. However, this formula does not always work well, since the model assumes that the share price changes continuously. This is not very realistic, since at the moments of uncertainty in the market the price of shares may be very volatile and often moves by jumps. Therefore the researchers and financial practitioners begun to examine the usefulness of models of the form $X_t = se^{X_t}$, where X_t is a Lévy process. The process X_t has independent and stationary increments and usually changes its value by jumps. The natural questions posed by economists and investors are as follows: what is the probability that next month the share price will rise, and how much it will increase; whether the price will reach a level required by the investor (at which the shares will be sold), and if so, what is the probability that it will take longer than the time t. At the moment the target level occurs and the shares are sold the process is terminated and this is modeled by killing the process after reaching or exceeding the required barrier.

Now, let us describe the same issue in the language of mathematics. We consider a Lévy process X_t with values in the multidimensional space $\boldsymbol{R}^d$ and kill it (stop it) at the moment of exiting a set D. We are interested in exact formulas of some quantities associated with this process, or at least their precise estimates. Such values for the killed process on exiting the set D are: $p_D(t, x, y)$ which is the transition probability that the killed process moves from x to y in time t; the distribution of $\tau_D(x)$ which is the first moment the process exits the set D starting from x (the distribution of the lifetime of the killed process); the Green function $G_D(x, y)$, that is the density of staying time of the killed process at y if the process starts from x; the average lifetime and the Poisson kernel, which is the density of the distribution of the process at the moment it leaves the set D.

For a general Lévy process and any open set D the above questions are very difficult to address. Therefore we intend to investigate Lévy processes with characteristic exponents possessing certain nice scaling properties. We will try to provide specific formulae (if possible) or precise estimates and asymptotics of transition probabilities, Green functions and Poisson kernels of such processes, killed on exiting a fixed set.

It turned out that the above described process of Samuelson S_t called the geometric Brownian motion is one of the coordinates of a Brownian motion in a hyperbolic space. Since α-stable processes, $0 < \alpha < 2$ proved to be, in many cases, better models than the Brownian motion, we try to explore the α-stable hyperbolic process.

Another class of processes that we want to study, are d-dimensional self-similar processes. The process X_t is self-similar in scale $\alpha>0$, if for some $c>0$ processes $c^\alpha X_{ct}$ and X_t have the same law. Such processes are used for modelling the physical and cosmological phenomena in which a suitable change in the scale of time and space does not change the law of the process. The trajectories of such processes are fractals. For multidimensional self-similar processes our objective is to obtain a description, analogous to the one, existing in one-dimensional case. Such a description would enable better understanding of the structure and behavior of self-similar processes.