Let us consider the following example of an underspecified ill-conditioned inverse problem. Our task is to recognize an animal based on the observation with binoculars. We can see only a tail because the rest of the animal is hidden in the bushes. Such a task will certainly have more than one solution. It is even difficult to estimate how many satisfactory solutions are there. Classical methods attack the problem from various points but they give us only a single solution, say a leopard. Stochastic sensitivity analysis methods show a distribution of the error of how much it is not a leopard, although it can still be a cheetah. The proposed methodology will give us all animals with tails sufficiently similar to the one seen through binoculars (featuring sufficiently small misfit from the observed one). Of course some methods solving such an ambitious problem already exist but they require enormous computational powers and long time. Our strategy will achieve its goal in remarkably cheaper possible way, i.e. significantly faster and by means of a middle-class equipment.

The above example states the characteristics of the proposed methodology in a simple way. However, its main application is definitely more commerce-oriented. Namely a real-world inverse problem solved by us (still more underspecified and still worse-conditioned than the recognition of animals based on tails) is related to oil and gas resource investigation. We exchange here looking through binoculars for drilling a borehole with simultaneous measurements of properties of rocks surrounding the borehole. A measuring device accompanies the drilling tool and emits electromagnetic waves with well-known characteristics. The waves are in turn scattered by rock layers and subsequently registered by receiver antennae of the measuring device. This way we obtain an image of electrical properties of the ground layers around the borehole. On this basis we would like to recognize the material forming the layers with the most careful attention paid to those containing interesting resources. Unfortunately the image obtained from the measurements gives us only a quite fuzzy imagination of the ground layer substance. Therefore, one has to solve an inverse problem consisting in 'fitting' the layer contents to the obtained image. And, as before, existing methods either simplify the reality concentrating on a single solution or find more solutions at huge cost.

The goal of the project is to design a strategy which allow for finding clusters of approximate solutions to parametric inverse problems, formulated as the minimization of misfit between the observed and simulated forward solutions. The strategy is dedicated to difficult, ill-posed problems, strongly motivated by the practice, in which misfit function is insensitive with respect to parameters to be found, over a sets with a positive measure. The approximation of such sets over which misfit reaches sufficiently low value as well will constitute the results of proposed inverse analysis. Such results may deliver much broader information to the experts in area (oil deposit investigation, medical diagnosis, flaw detection, etc.), than the single solutions found by the standard stochastic methods or the steepest descent methods of convex optimization.

The proposed method intelligently associates evolutionary-type random search on several accuracy levels with local search methods. The aim of the random search is finding sets containing interesting solutions. On the worst accuracy level the search is quite chaotic, but it becomes more and more focused with increasing accuracy. After finding a set possibly containing a solution the strategy executes the local search which quickly fixes the exact position of the solution (or states the lack of the solution).

The main computational burden in the considered problem is the computation of the misfit between a current solution and the observed data. The proposed methodology reduces this cost twofold. First, an appropriate configuration of the several-level random search and the local search aims at the maximal reduction of the considered solution set. Second, the actual computation is executed in parallel by means of modern, very efficient computational algorithms.