The proposed project is in the mainstream of current research on the border of algebraic geometry and arithmetic. Algebraic geometry studies sets of solutions of polynomial equations. It has close ties on the one hand with number theory (e.g., proof of Fermat's Last Theorem) and on the other hand with physics (string theory, quantum field theory, mirror symmetry). Contemporary algebraic geometry provides a uniform language in which one can talk about these different subjects. This caused a surprising flow of ideas between these superficially unrelated subjects. In the proposed project we study both geometry of varieties defined over complex numbers and over fields of positive characteristic, mixing the points of view of mathematical physics and number theory. The main motivation behind these studies is their possible application to number theory, topology, differential equations and in mathematical physics.

For example, one of the proposed problems pertains to general differential equations on complex algebraic varieties and the attempt to classify them. In simple cases (the so called regular singular case), holomorphic (or more precisely algebraic) differential equations can be classified topologically by means of the associated representation of the fundamental group (the so callled monodromy reppresentation). The classifying space for representations of the fundamental group has a simple algebraic structure that can be obtained by writing down the group using its generators and relations between them. However, the holomorphic change of differential equations leads to other algebraic structures that this naive one and it has been well understood only in the curve case. The problem we consider is about existence of such a structure in the general case, when not only the description of the solutions by means of monodromy is insufficient, but also the dimension of the variety is arbitrary. The main motivation behind studying this problem comes from arithmetic. More precisely, it comes from Deligne's theorem about finiteness of *l*-adic sheaves with bounded ramification on varieties defined over finite fields. The proof of this theorem uses some deep results of Lafforgue about Langlands correspondence on curves. In case, we consider the analogous problem on varieties defined over fields of positive characteristic, it begins to have deep relations to certain conjuctures on D-modules on algebraic varieties. In particular, the solution of such a problem in positive characteristic woyld give the proof of Gieseker's conjecture saying that vanishing of the fundamental group implies triviality of all D-modules on such a variety.

The remaining problems are somewhat more dificult to state in an elementary fashion. All of them are related to the study of representations of the fundamental group of an algebraic variety and to geometry of varieties defined over fields of various characteristics. They are also deeply related to arithmetic. They pertain to a generalization of the non-abelian Hodge theory to positive characteristic and to the p-adic case, the study of homotopy theory of varieties in positive characteristic, the study of monodromy group via log geometry and the study of p-adic Langlands correspondence by means of the newly created perfectoid spaces.

The techniques used in solution of the above problems have already had many applications. The non-abelian Hidge theory is an indespensible tool in the geometric Langlands program and it appeared in the quantum field theory in the work of Witten and his collaborators: Kapustin, Gukov and Frenkel. Homotopy theory in positive characteristic plays an important role in the p-adic Hidge theory and solutions of the problems we pose should allow to understand the basic ingedients describing the fundamental group of an algebraic variety in general. The study of the monodromy group by means of log geometry has already had some important applications in the Gross-Siebert program on the mirror symmetry. The solution of the proposed problems should allow for a systematic treatement of their theory.
Finally, perfectoid spaces allow one to transfer some results between characteristic 0 and characteristic p. But so far the possible scope of their applications is not really well understood which motivates our studies.

Even partial solution of the proposed problems should give some important results in various parts of the Langlands program. In general, the program has a potential to be applied in various geometric and arithmetic problems, even so elementary as the study of possible singularities of the plane curves.