

Model theory describes classical mathematical structures (e.g. natural numbers with addition, real numbers with addition and multiplication) in an abstract, but structural manner (unlike set theory). In this context, those structures are called models (especially when we understand them as models of some particular mathematical theories).

This allows us to choose within them certain subsets, using formulas (e.g. the set of even numbers, the set of real numbers greater than 3). Such sets are said to be definable.

“Naturally” occurring models often have very little symmetry, but as it turns out, there are some constructions which allow us to extend them to bigger structures, including very large and with very rich symmetries (so-called monster models), while preserving many important properties of the ones we have begun with. In particular, definable sets, interpreted in larger structures, behave the same, in a certain well-defined sense. Similar things happen (although to a lesser degree) to the intersections of collections of definable sets (those intersections are said to be type-definable sets).

In general, those sets which can be seen in different models are called invariant, and it so happens that they are exactly those sets which are symmetric in monster models.

Apart from sets, we also consider partitions of models, and among those partitions we distinguish those which are invariant in an analogous manner. As it turns out, the properties of some particular partitions are of great importance in classification of mathematical theories, which allows us to say that some theories are more “wild” (like the theory of whole numbers with addition and multiplication), while others are more “tame” (like the theory of whole numbers with just addition, and to a lesser degree the theory of real numbers with addition and multiplication). Moreover, the research in this direction has shown a variety of hitherto unknown connections of model theory and other branches of mathematics, while the “spaces” obtained by those partitions can often be identified with classically occurring mathematical objects.

The goal of this project is to understand the nature of those partitions, and the possible degree of their complexity. This is a relatively new line of study, and the current knowledge is highly unsatisfactory, which this project should improve.