Rings are important objects in mathematics. A ring is a set, together with two binary operations: addition and multiplication. These operations have properties of the usual addition and multiplication. The only difference is it might not be possible to divide by elements different from zero. If this is also possible, then we call the ring a field. The best known example of a ring (which is not a field) is the set **Z** of integers. If a ring contains a set, which is a field (with the same operations), then we call it an algebra over this field. The set *K[X]* of polynomials with coefficients in a field *K* is an example of an algebra.

Rings are often studied via their categories of modules. A module over a ring *R* is a set *M* with a binary operation called addition, such that we can multiply the elements of *M* by the elements of *R*. If *n* is a positive integer, then the set {0, 1, ..., *n*-1} with the addition modulo *n* and the multiplication by the integers modulo *n* is a module over **Z**. If *R* is an algebra over a field *K*, then we call a module *M* finite dimensional, if there exists finitely many elements $m_1, ..., m_d$ of *M* such that every element of *M* can be obtained from these elements by performing addition and multiplication by elements of *K*. We call the smallest *d* with this property the dimension of *M*. If a sequence $m_1, ..., m_d$ realizes this minimum, then we call it a basis of *M*. Note that the algebra *R* is a module over itself, hence we may use this language with respect to *R* as well.

Fix an algebra *R* of dimension *e* over a field *K*. If *d* is a positive integer, then we can associate with a module *M* a sequence of ed^2 elements of *K*, which describes the multiplication of the elements of *M* by the elements of *R*. The set of all sequences, which can be obtained in this way, is described by polynomial equations within the space of all sequences of length ed^2, hence is an affine variety in the sense of algebraic geometry. This set is called the variety of *d*-dimensional *R*-modules and denoted by $\mathrm{mod}_R(d)$. A sequence associated with *M* is not uniquely determined - it depends on the choice of a basis of *M* (on the other hand, we keep a basis of *R* fixed). We denote by O_M the set of all possible sequences we may get for *M* and $\mathrm{cl}(O_M)$ denotes the closure of O_M, which roughly speaking is the set of points which are close to O_M. The set $\mathrm{cl}(O_M)$ is again an affine variety.

The main aim of our project is to study geometric properties of varieties $\mathrm{mod}_R(d)$ and $\mathrm{cl}(O_M)$. For example, if *x* is a point of an affine variety *X*, then one defines the tangent space T_xX to *X* at *x*. This construction generalizes the construction of the tangent line and the tangent plane known from classical geometry. A point *x* is called a non-singular point of *X* if, intuitively speaking, the space T_xX is as big as *X* in a neighbourhood of *x* (formally speaking, the dimension of T_xX equals the dimension of *X* at *x*). An important problem is to study, when a given point is non-singular and what types of singular points we may encounter.

The problems we want to study are of geometric nature. However, we plan to investigate them using the interpretation of the points of the studied varieties as modules over algebras. This allows us to use methods from representation theory, which are often of homological nature. Homological properties of modules often correspond to geometric properties of the corresponding points. This observation has been used previously by many authors in geometric studies. We plan to continue this line of research and develop new methods, which should allow us to get deeper results.

We also plan to study homological problems connected with algebras. An important homological invariant associated with an algebra is its bounded derived category. We will study the bounded derived categories for the derived discrete algebras (i.e. the algebras without "continuous families" of modules in their derived categories). Our aim will be to prove existence of so-called derived Ringel-Hall polynomials.